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LEADING ARTICLE

Induction versus Popper: substance versus semantics

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This article reviews concepts of classical logic and induction, with special attention to the controversies surrounding Popperian claims that induction is impossible and does not exist. I argue that some of the controversy is semantic, and hence Popperian criticisms of induction must be translated carefully into ordinary language to be appreciated by inductively oriented epidemiologists. With this translation, the substance of the debate is not whether induction is possible (it is) or exists (it does), but whether and how we should employ probabilistic reasoning about hypotheses in epidemiological inference.

Keywords Induction, inference, logic, Popper, probability, statistics

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The last several decades have witnessed some lively arguments between Popperian and non-Popperian epidemiologists, with special focus on the roles of deduction and induction in observational studies.[1–5] In a simplified form, a good part of the controversy could be summarized as follows: While all parties agree that classical deductive reasoning is necessary for sound inference, it seems that Popperians have maintained it is sufficient as well, while non-Popperians have maintained that non-deductive methods are also necessary. Popperian views have also been promoted in biostatistics,[6,7] along with reservations about the sufficiency of these views.[8,9]

To a certain extent, these controversies reflect debates in the philosophy of science.[10–12] Nonetheless, I will argue here that, within epidemiology, the controversy has become clouded by the diversity of processes that have been termed 'induction'. In particular, I will consider some of the most controversial assertions of Popper and his followers, to the effect that induction is impossible and that induction does not exist (ref. 10, pp.1013–15). I believe these assertions have been widely misunderstood or ignored because their meaning in Popperian language corresponds poorly to their ordinary language meanings. Furthermore, the meaning of 'induction' in these statements does not correspond at all to its meaning in modern Bayesian philosophy.[12] Recognition of these semantic discrepancies would, I believe, remove the cognitive dissonance and reduce the sense of controversy surrounding the assertions.

The controversy surrounding inductive reasoning may be further aggravated by some surprising vagaries and misunderstandings surrounding concepts of deductive inference, which is supposed to be the least controversial element in scientific reasoning (it is universally held to be necessary; only its sufficiency appears to be in question[11]). Therefore, I will begin this essay by reviewing deductive concepts.

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Deduction

The reader turning to an ordinary language treatise will probably find definitions resembling these, taken from the *Shorter Oxford Dictionary*:[13]

Deduce. 5. To draw as a conclusion *from* something known or assumed; to derive by reasoning; to infer.

Deduction. 5. The process of deducing from something known or assumed; *spec. in Logic*, inference by reasoning from generals to particulars.

The definition of deduction as 'reasoning from generals to particulars' is often quoted. Nonetheless, it is not consistent with its definition in formal studies of logic (metalogic)[14,15] or in everyday usage; that is, deduction need *not* proceed from generals to particulars. As an example, from the premise (assumption) that 'Employees 1237 and 4291 developed mesothelioma because of their asbestos exposure' we may deduce that 'Asbestos sometimes causes mesothelioma'. In ordinary English, the premise is not general (it refers to two specific employees) and the conclusion is not particular (it refers to no one in particular). Of course, the premise contains the idea that asbestos can cause mesothelioma. But this fact underscores the nature of deductive arguments: The premises already contain the conclusion in a logical sense; deductive arguments only transform the information contained in the premises.[16]

Turning to a dictionary of philosophy,[17] we find

deduction: (Lat. deductio, a leading down) Necessary analytical inference. (a) In logic: inference in which a conclusion follows necessarily from one or more given premises. Definitions given have usually required that the conclusion be of lesser generality than one of the premises, and have sometimes explicitly excluded immediate inference; but neither restriction fits very well with the ordinary actual use of the word. (b) In psychology: analytical reasoning from general to particular or less general. The mental drawing of conclusions from given postulates.

Part (b) parallels the Oxford definition, but part (a) offers something different: The notion of *logical necessity* of a conclusion given the premises. This notion is at the core of concepts of valid reasoning.

In modern studies of logic, the word 'deduction' takes on another, more precise meaning: A *deduction* or *derivation* refers to a form of argument in which conclusions are derived from premises using explicit rules (sometimes called deductive forms or formation rules[14,16]) for deriving new statements from previously given statements. In what follows, I will illustrate and use this definition of deduction.

An argument *form* or *mode* is an abstraction in which letters stand for arbitrary statements.[16,18] In the following presentation I will use just two letters, H and B, which may be thought of as standing for a hypothesis H and a possible observation or association B. The statement form 'H implies B' (equivalent to 'if H then B' and 'B if H') will be pivotal, and may be thought of as an assertion that the hypothesis H predicts the observation B. In this context, the statement form 'not H' may be thought of as an assertion that the hypothesis H is false, and the statement form 'not B' may be thought of as an assertion that something different from B was observed, or that B is false.

The notion of logical necessity arises when one recognizes that certain forms of deduction are *logically* or *deductively valid*, in that it is impossible for both their premises to be true and their conclusion to be false.[16,18] In such forms, the premises are sometimes said to *entail* the conclusion.[17] One classic valid form is *modus ponens*,

Premise 1:	H implies B
Premise 2:	H
Conclusion:	B

This form is valid because no substitution of statements for H and B can make 'H implies B' true, H true, and B false.[16,18] Another classic valid form is *modus tollens*,

Premise 1:	H implies B
Premise 2:	not B
Conclusion:	not H

Falsificationists (such as Popper) take this form as the centerpiece of scientific reasoning, in that it is the basis for *refutation*: If H is an hypothesis, B is a prediction of the hypothesis (so that H implies B), and B turns out to be wrong, then by *modus tollens* H must be wrong as well.

Deductive fallacies are forms of argument that are not deductively valid, in that the premises can be true and the conclusion false. A classic example is the *Fallacy of Affirming the Consequent*,

Premise 1:	H implies B
Premise 2:	B
Conclusion:	H

Unfortunately, this fallacy embodies an all-too-common approach to 'scientific' inference: A researcher will note that a hypothesis H (often his or her favourite) implies a prediction B, observe that B is indeed what has been observed, and conclude that H must be correct. The fallacy appears to be especially rampant in epidemiology. Typically, H is a causal hypothesis (e.g. 'silicone implants do not cause scleroderma') and B is an analogous statistical observations (e.g. 'silicone implants are not significantly associated with scleroderma'). Even if one incorrectly ignores all the biases and random errors that are inevitably present and undermine the premise 'H implies B', it is still a logical fallacy to infer H from B.[16,18]

The Fallacy of Affirming the Consequent remains a logical fallacy even if B represents all observations that have been made to date. For example, the hypothesis H that 'Cigarette smoking causes lung cancer' successfully predicts nearly all (if not all) the epidemiological observations made to date (including so-called 'anomalies' often raised as inconsistent with the hypothesis, such as the site of tobacco-associated lung tumours[19]). Nonetheless, this overwhelming predictive success of the hypothesis does not provide a deductively valid proof of the hypothesis; to assert otherwise is an example of the above fallacy. It remains *logically* possible that some key confounder has been overlooked. Such a hypothesis sounds utterly implausible; deductive logic involves no consideration of plausibility, however, and so to many observers appears incomplete as a basis for scientific inference.[11,12]

When H is a scientific law or theory, there will *always* be other hypotheses that predict the observations B and contradict H. In other words, every set of observations B will have many conflicting explanations. It is a fact of logic that deduction alone is insufficient to identify the correct explanation (hypothesis) from among those that correctly predict what has been observed.[10–12,20] In other words, the scientific laws that lead to a series of events cannot be validly deduced from observations of the events; they are deductively *non-identifiable* or *non-provable*, at least if one does not make assumptions that are themselves non-identifiable, such as absence of residual confounding.[21] Hume recognized this problem over 250 years ago,[20] and it has since become the starting point of many philosophies of science;[10–12] for example, Popper devoted much of his career to emphasizing the problem and to proposing how to proceed with science once non-identifiability was accepted.[10] He concluded that reasoning must work within the limits of deductive methods, whereas many other philosophers refuse to accept such circumscription of science.[11,16]

Induction

The issue of how to proceed with science in the face of non-provability of theories has been the source of endless controversy, most notably the controversy surrounding concepts of induction. The literature on this topic is vast, and there is no space here to even outline the classic treatments of induction, such as those of Bacon, Whewell, Mill, Peirce, Keynes, and Carnap; even an outline of all the definitions of induction that have been offered would be lengthy. Salmon[16] and Hempel[18] give elementary introductions to the key issues, while Kyburg[22] and Cohen[23] provide advanced and detailed treatments.

The Fallacy of Affirming the Consequent has at times been labelled as induction. It has been lamented that scientists often reason with this fallacy,[24] so that if one labels the fallacy as 'induction', then induction not only *is* possible, but exists and is common. Of course, the label of 'induction' justifies neither the fallacy nor induction; rather, it discredits induction as a valid form of reasoning.

Ordinary language dictionaries offer more general definitions of induction. In the *Shorter Oxford Dictionary*,[13] we find

Induction. 7. *Logic*. The process of inferring a general law or principle from the observation of particular instances (opp. DEDUCTION). 1553.

The Fallacy of Affirming the Consequent fits this definition whenever H is a general hypothesis and B is a particular fact or collection of facts predicted by H. More often, however, *induction* refers to the following creative process: An unexpected association is observed (e.g. a *positive* association of beta-carotene with lung cancer in a randomized trial). This finding causes observers to invent hypotheses that explain (predict) the association (e.g. beta-carotene protects newly malignant cells from destruction by the immune system). Observers might not even think of these hypotheses without the unexpected observation. In this very real sense, data can generate hypotheses.

The process just described is sometimes called *abduction* or *retroduction*.[25] Cognitive psychology, as well as common sense, tells us that some sort of creative process takes place in people's minds in which general hypotheses are generated in response to particular observations. Thus, for those who identify induction with retroduction, the important question is not whether inductive thought processes exist; they do exist, as exemplified by the inventive process as well as many cognitive illusions.[26,27] Rather, the question is whether they can be constrained to yield true hypotheses from true premises, and so provide valid *deductions* of hypotheses.

As if to answer this question, the entry for 'induction' in the *Oxford Dictionary of Philosophy*[15] begins by stating that 'The term is most widely used for *any* process of reasoning that takes us from empirical premises to empirical conclusions supported by the premises, *but not deductively entailed by them*' (emphasis added). The Fallacy of Affirming the Consequent fits this description, at least if one allows that this description does *not* exclude the possibility that 'the process of reasoning' may involve theoretical premises (such as 'H implies B'), as well as empirical ones. The combination of this fallacy with retroduction, in which one invents a hypothesis to explain the observations and then accepts the hypothesis *because* it explains the observations, also fits this description of induction.

Popper on induction

At the outset of *The Logic of Scientific Discovery*,[28] Popper noted that 'It is usual to call an inference 'inductive' if it passes from *singular statements* … such as accounts of observations or experiments, to *universal statements*, such as hypotheses or theories.' This definition is in good accord with ordinary language definitions, and Popper's chief focus in the book was to argue that such inferences could not be justified in any deductively valid or other compelling manner. His assertion was neither as novel nor as controversial as often supposed, as evidenced by the above quote from the *Oxford Dictionary of Philosophy* and this passage from Bertrand Russell (1903), quoted by Jacobsen:[29] 'I do not distinguish between inference and deduction. What is called induction appears to me to be either disguised deduction or a mere method of making plausible guesses.'

At later points, however, Popper defined inductive inference as 'inference from repeatedly observed instances to as yet unobserved instances' (ref. 10, p.1014). This definition differs from the dictionary definitions in two prominent ways: First, in the notion of repetition of the observations; second, in the notion that the inference is to 'as yet unobserved instances', rather than to general hypotheses or to empirical conclusions that may already have been observed. The definition remains somewhat vague, however, in part because the vagueness of the word *infer*. Ordinary language dictionaries (e.g.[13]) define 'to infer' as nothing more than drawing a conclusion. With this definition, inference (the process of drawing a conclusion) may involve anything: Deductive arguments, looking at graphs and tables, inventing hypotheses that predict what was already observed (retroduction), consulting Ouija boards, or even significance testing. None of these activities need lead to valid conclusions, but they exist and in fact are often used as part of processes in which unobserved instances (events) are predicted from observed (and perhaps repeated) instances.

Within a few paragraphs of the above definition, Popper clarified his views by stating that 'I hold with Hume that there simply is no such logical entity as an inductive inference; or, that all so-called inductive inferences are logically invalid' (ref. 10, p.1015). This is hardly a controversial assertion, given that none of the earlier definitions of induction say anything about logical validity, except to note that it is *not* a part of the definition of induction. Within a few more paragraphs, however, Popper made a highly controversial assertion: 'I hold that neither animals nor men use any procedure like induction, *or any argument based on repetition of instances.* The belief that we use induction is simply a mistake.' Then, shortly thereafter, he claimed that *'Induction simply does not exist,* and the opposite view is a straightforward mistake' (emphases added). This assertion did set Popper apart from his predecessors. For example, Hume never claimed that induction was impossible or non-existent; rather, he argued that it was a logically unfounded habit of the human mind.[20]

Perhaps you never went back to a particular restaurant because every time you went there you had a bad meal; or you refused to revisit a person or place because every time you visited you had a bad time; or perhaps you refused to invest money in something because every time you did you lost money. If so, you may feel (as I do) that the claim that we never use 'any argument based on repetition of instances' is superficial and absurd, and will perhaps wonder (as I have) how Popper attracted so many devotees (and so many intelligent ones at that). There is, however, a reasonable explanation for Popper's claim. It is this: Popper meant that we never use any argument based on observed repetition of instances that does not *also* involve a hypothesis that predicts both those repetitions and the unobserved instances of interest.[25]

More precisely, Popper maintained that the act of moving from the observed to the unobserved involves two steps: First, formulation of a hypothesis that entails or predicts (and so is corroborated by) the observed instances; then, use of this hypothesis to *deductively* predict the as-yet unobserved instances. This process is the falsificationist version of 'hypothetico-deductive method'.[15] The process could just as well be called 'induction', but the word 'induction' may refer only to the hypothesis-formation step. In any case, under the preceding explanation, the phrase 'induction does not exist' could be translated into common terms as 'There is never any direct logical relation between the observed and unobserved instances; all relations between them are indirect, via a hypothesis that entails all the instances.' The correctness of this assertion remains controversial, but the very need for translation reveals a language gap that has, I believe, aggravated the controversy about the role of 'induction' in epidemiology.

Another way to interpret Popper's claim so that it makes sense is as an answer to the following question: Is there any method of reasoning that will, *with certainty*, lead us to the correct explanation of our observations? Such a method would, of course, be invaluable if it existed, but Popper's writings make clear that he regarded such a method as impossible and (hence) non-existent.[10,28] As discussed earlier, there appears to be little or no controversy on this point. Because of the non-identifiability of theories, there indeed can be no such method.

An open question is whether some systematic approach to the cycle of hypothesis generation and observation will eventually converge to correct hypotheses. The belief that 'the scientific method' is just such an approach seems implicit in much scientific writing. As evidenced by the literature cited here, however, there is much less agreement on what constitutes 'scientific method' than commonly believed. Some writers have even questioned whether any scientific methods need necessarily lead to correct theories[30] or are essential to the growth of knowledge.[31]

Statistical induction

Nearly every type of statistical approach has at times been called 'inductive'. RA Fisher referred to his significance-testing procedures as methods for 'inductive inference';[32] Neyman referred to his hypothesis-testing procedures as methods for 'inductive behavior';[33] R von Mises referred to his objective Bayesian approach as an 'inductive science';[34] and DeFinetti referred to his subjective Bayesian approach as 'inductive reasoning'.[35] Despite the diversity of approaches represented by these authors, their usage is defensible.

Consider as an example the problem of estimating a prevalence proportion P in a large population based on a random sample of size N. If A cases are observed, both the 'straight rule' of induction[22] (enumerative induction[16]) and Fisher's maximum-likelihood approach tell us to take A/N as our estimate of the population prevalence. These rules are mechanical algorithms for making choices among competing hypotheses about the prevalence P. Nothing in the deductive derivation of A/N as the maximum-likelihood estimate dictates its choice, and none of the usual reasons given for preferring A/N (e.g. minimum-variance unbiasedness) is compelling.[12,36] That is, there are non-deductive and non-compelling elements in the justifications for their use. In this regard, they fit the usual description for inductive procedures.

In light of Bayesian usage, to claim that 'no probability statement … is an inductive generalization from a sample to a population' (ref. 25, p.156) is wrong, simply because 'inductive generalization' has many meanings, including some in which it *does* refer to a probability statement about a population that is computed from a sample. Let P_L and P_U be the 2.5^{th} and 97.5^{th} percentiles of a Bayesian posterior distribution[12,35] for the prevalence. We then have the Bayesian 'inductive generalization'

$$\Pr(P_L < P < P_U) = 0.95 ,$$

which is a probability statement about the population prevalence P deduced from the sample, the likelihood or sampling model, and the prior distribution.

Deductive induction

To summarize up to now: Modern philosophers have had to come to terms with the problem of non-identifiability (non-provability) of general laws. In particular, a general law or theory cannot be validly deduced or proven to be true no matter how many of its predictions are borne out. This means that induction, *as commonly defined*, cannot be put on a deductively valid logical foundation. But there are other processes called induction that refer to deductively valid arguments and thus have a valid foundation. I will refer to deductively valid arguments that are given inductive labels as *deductive inductions.*

Mathematical induction[14,15] (also known as *proof by recursion*[17]) may be the best known form of deductive induction. Suppose S(n) is a statement about an unspecified integer n. (An example is Stein's theorem: 'Given random samples from n + 2 populations, there is an estimator for the vector of population means that has smaller mean-squared error than the vector of sample means.') The form of argument is

Premise: S(1)
Premise: S(n) implies S(n + 1)
Conclusion: For all n, S(n)

In English: If the statement about an integer n is true when 1 is substituted for n, and is true for n + 1 whenever it is true for n, then it is true for every integer. This form of argument does not fit common definitions of induction because the second premise is a general statement, so the reasoning is from one specific and one general premise to a general conclusion.

Mathematical induction is only occasionally used in epidemiology, and then only in methodology. There are, however, other forms of deductive induction based on probabilistic reasoning that are sometimes called *probabilistic induction*.[12,36,38] As an example, if (as we should) we consider the prior distribution as part of the assumptions underlying a statistical analysis, so that the prior is recognized as potentially incorrect and open to criticism (like the assumption of random sampling), then Bayesian estimation is a purely deductive process.[37,38]

Unfortunately, most people do not comprehend probabilistic reasoning in general and probabilistic induction in particular.[26,27,39] The topic of probabilistic induction also raises many semantic and technical issues, as well as philosophical disputes;[10–12,22,23,35–42] for example, some pivotal writers write of these forms as if they were distinct from classical deductive forms,[35] even though they are theorems of probability and so are in fact deductively valid arguments.[38] Because an accurate discussion of probabilistic induction requires a background of probability logic,[12,22,23,39] I have deferred this discussion to a companion paper.[38]

Finally, it is perhaps ironic that the refutational approach associated with Popperian philosophy is an integral component of a deductively valid process known as *eliminative induction*:[15] If we *assume* as a deductive premise that one of the competing hypotheses H_1, ..., H_n is correct, and proceed to test them all against observations until only one remains unrefuted, then we may deduce that this remainder is correct. The assumption can be satisfied by insuring that the list H_1, ..., H_n exhausts all logical possibilities. Eliminative induction is a staple of classic outbreak investigations; for example, to search for necessary causes of a diarrhoea outbreak at a picnic, the investigator attempts to compose an exhaustive list of exposures, and then eliminates those exposures that were not present in all cases. The assumption that the list is exhaustive is also subject to refutation by the process (it would be refuted if all the hypotheses in the list were refuted).

Discussion

I have reviewed some basic concepts of logic and induction, and concluded that part of the controversy surrounding inductive reasoning in epidemiology stems from vagueness and variation in definitions of inductive concepts. With this view, the controversy between Popperians and non-Popperians that is labelled 'inductive' is *not* about whether 'induction is impossible and does not exist,' as Popper and some followers claim, but whether any of the processes that have been labelled as 'induction' can be recommended for epidemiology.

There is no controversy about mathematical or eliminative induction: They are valid deductive tools and so can be used with the usual caveat that their conclusions depend on the correctness of their assumptions. There is also no controversy that 'induction', in the sense of claiming that a hypothesis is true because its predictions are borne out, is a *deductive* fallacy—although there is much controversy surrounding the nature of the support provided for a theory by correct predictions.[10,12,22,23,34–42]

I do not think anyone seriously doubts that 'induction' in the sense of retroduction (inventing a hypothesis to explain unexpected observations) is an integral component of creative scientific activity. It is not its existence or importance, but the *origin* of such invention that is contested by Popperians. Standard descriptions of the process assert that data anomalies are the initial cause of hypothesis generation, provoking an explanation from the observer.[43] In contrast, Popperian descriptions point out that an observation can be anomalous only in relation to a theory; thus, to 'observe an anomaly' the observer must have had some prior expectation that conflicts with what was observed; this prior expectation could only arise from some previously accepted theory.[25] It is this clash of observations with expectations that causes the observer to generate a new hypothesis to explain the anomaly.

My opinion here is akin to Susser's,[44] in that it seems of little importance whether we label the creative process of hypothesis generation as inductive generalization (although this label invites confusion with other processes), abduction, retroduction, or conjecture. Nonetheless, I think that Popperians are correct to emphasize that data anomalies can be defined as anomalies only in relation to prior expectations, and hence prior expectations must play a key role in observation and hypothesis generation.

I diverge from Popper and agree with Susser[44] and Jacobsen[45] in that I believe Popper's philosophy fails to successfully address most statistical issues that vex epidemiology.[42] Those issues can, however, be addressed through the use of prior expectations in subjective Bayesian analysis—which, though deductive in character, is sometimes called 'probabilistic induction'.[12,36] Such analyses logically complement criticism and testing of the hypotheses from which the prior expectations are derived by providing probabilistic measures of support for hypotheses.[37,38] Even Popper used such measures in his arguments against induction;[40] controversy arises only over the utility of the measures and whether they are anything more than deductive in nature.[12,22,23,35–42]

To summarize, I believe the following questions underlie the most substantial (as opposed to semantic) disagreement between Popperian and non-Popperian epidemiologists: Must all epidemiologic inference be constrained to follow classical deductive forms, and thus exclude anything resembling confirmation of hypotheses (as some Popperians have maintained)? Or is there a role for positive assertions about hypotheses, even if these assertions are only probabilistic (as all Bayesians and many non-Bayesians[32] have maintained)? The answers have important implications for epidemiological analysis—for example, in deciding whether epidemiological statistics should move toward a Bayesian paradigm.[38] Unfortunately, agreement on the answers may not be close at hand.

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